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Critical phenomena in fractal spin systems

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Abstract. Classical ferromagnetic spin systems on self-similar fractal lattices are studied. By a variation of Simon's argument, it is shown rigorously that the critical decay exponent Δ (which is characterised by $\langle\varphi_x\varphi_y\rangle \leq \mathrm{dist}(x, y)^{-\Delta}$ at the critical point) satisfies the bound $\Delta \leq Q$, where Q is the connectivity.

We also discuss some general features of the phase transitions and critical phenomena.

1. Introduction

Fractals [1], as a new universality class of geometric objects in nature, have been attracting considerable interest. In particular, non-random fractals, constructed via simple recursion relations, are useful as textbook examples for the study of physics in an environment which lacks translation invariance but possesses self-similarity. Such a situation, common in various random systems, might also take place in the structure of spacetime itself if we go beyond the Planck length scale.

In fact, spin systems, field theories, quantum mechanics, diffusion processes, electric networks and many other interesting physical systems on fractals have been studied (see [2-6] and references therein). There, one of the main interests is to single out and calculate a certain simple quantity (dimension) governing the universal behaviour of the relevant system.

In the present paper, we concentrate on the classical ferromagnetic spin systems (or, equivalently, the scalar field theories with UV cutoff) defined on fractal lattices. (We call them 'fractal spin systems'.)

First we discuss some of the general features of phase transitions and critical phenomena in fractal spin systems. Then we study decay properties of the system at the critical point and prove a rigorous inequality for the exponent of the power-like decay. It can be regarded as a fractal version of the Simon inequality [8]. In the derivation of this inequality, we are naturally led to the notion of the *connectivity*, which is very similar to that in [3].

We would like to stress that such an inequality plays a crucial role in determining the long-range scaling behaviour of spin systems near the critical point and the basic scaling properties of the (possible) continuum-limit field theories. We also note that this is among the few rigorous and concrete results obtained for physical systems in general fractals, including the infinitely ramified ones.

In § 2, we define spin systems on fractal lattices, establish their high-temperature behaviour, discuss basic features of their phase transitions and briefly describe our main inequality. Section 3 is devoted to the detailed description and proof of our inequality.

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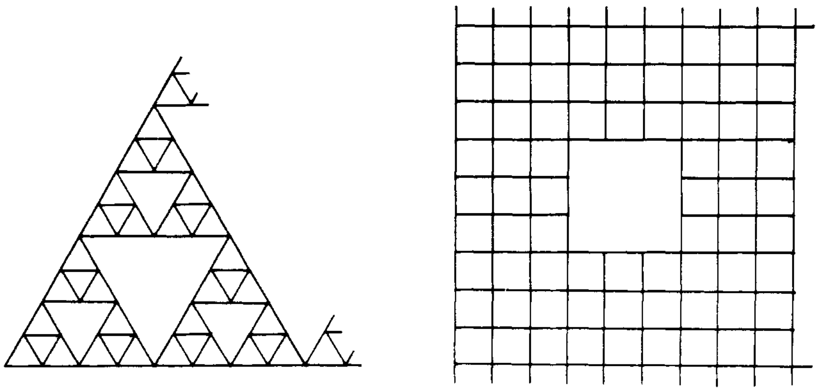
2. Phase transitions and critical phenomena in fractal spin systems

2.1. Definitions

Let Λ_s be a set consisting of a countably infinite number of *sites*, $x, y, \ldots \in \Lambda_s$. We associate with Λ_s a set of *bonds* Λ_b which consists of unordered pairs of sites (x, x'), $(y, y'), \ldots \in \Lambda_b$. To each site x, the number of the bonds in Λ_b including x is uniformly bounded by a constant z. We assume that $\Lambda = (\Lambda_s, \Lambda_b)$ defines a *connected non-random fractal lattice* which is self-similar in the large length scale. The examples are almost all of the known fractal lattices, including p-dimensional Sierpinski gaskets and carpets, constructed of bonds of unit length [1-6] (figure 1). Our precise assumptions will be described in § 3 (see F1-F7).



(*a*) (*b*)

Figure 1. (*a*) Sierpinski gasket. $d_{FRAC} = \ln 3/\ln 2 \simeq 1.585$, $Q = 0$ and $d_{SPEC} = 2 \ln 3/\ln 5 \simeq 1.365$. (*b*) Sierpinski carpet. $d_{FRAC} = \ln 8/\ln 3 \simeq 1.893$, $Q = \ln 2/\ln 3 \simeq 0.631$ and D_{SPEC} is conjectured [6] to lie between 1.72 and 1.86.

Spin systems (or field theories) on Λ are defined in the usual manner by associating *spin variables* $\varphi_x \in R^N$ ($N = 1$ or 2) to each site $x \in \Lambda_s$, and a pairwise *ferromagnetic interaction* to each bond $(x, y) \in \Lambda_b$. Our Hamiltonian $\mathscr{H}$ and the corresponding thermal expectation are†

$$\mathscr{H} = -\sum_{(x,y)\in\Lambda_b} \sum_{i=1}^{N} \varphi_x^{(i)} \varphi_y^{(i)}$$

$$\langle\ldots\rangle = Z^{-1} \int \prod_{x\in\Lambda_s} \mathrm{d}\nu(\varphi_x)(\ldots)\,\mathrm{e}^{-\beta\mathscr{H}} \qquad \beta > 0, \langle 1\rangle = 1 \tag{2.1}$$

where the single-spin distribution $\mathrm{d}\nu(\varphi)$ is assumed (from technical considerations) to be in the Brydges-Fröhlich-Spencer class [9], i.e. measures written as $\mathrm{d}\nu(\varphi) = \exp(-V(\varphi^2))\,\mathrm{d}\varphi$ with $V''(t) \geq 0$ for $t \in [0, \infty)$, and their well defined limits. In particular, the class contains the Ising model, the classical XY model and one- and two-component lattice φ^4 field theories‡.

† More precisely, we define $\langle\ldots\rangle$ as an infinite volume limit of a sequence of finite volume expectations.

‡ In the following, we describe our results for the single-component spin systems. The extensions to the two-component ones are straightforward.

2.2. *High-temperature region*

First, we consider the high-temperature region where β is sufficiently small. Let us introduce the *graph-theoretic distance* $\mathrm{dist}(x, y)$ which is defined as the minimum number of bonds in Λ_b that one needs to connect two sites x and y in Λ_s. Then we can easily state the following.

Theorem 1. The two-point function of the system satisfies the following bound for all $x, y \in \Lambda_s$ and $\beta < \beta_0$:

$$\langle \varphi_x \varphi_y \rangle \leq \text{constant} \times \exp(-\ln(\beta_0/\beta)\, \mathrm{dist}(x, y)) \tag{2.2}$$

where $\beta_0 = 1/cz$ with $c = \int d\nu(\varphi)\varphi^2 / \int d\nu(\varphi)$. Therefore it exhibits exponential decay.

Proof. By a standard high-temperature expansion type argument [10, 11], one can bound the correlation function in terms of the free random walks on the fractal lattice. Equation (2.2) then follows by an elementary estimate of the number of random walks.

Remark. The above theorem applies to a more general class of spin systems (see [10, 11]).

Note that $\mathrm{dist}(x, y)$ is *one* of the natural notions of distance we can equip to the fractal lattice Λ. The above theorem shows that the present one is a suitable distance for spin systems (at least) in the high-temperature region.

2.3. *Phase transitions*

It is believed (and partially proven rigorously) that a ferromagnetic spin system in the ordinary hypercubic lattice undergoes a *phase transition* if the dimension d satisfies $d > 1$ (spin system with discrete symmetry) or $d > 2$ (spin system with continuous symmetry). In particular, it exhibits the *long-range order*

$$\langle \varphi_x \varphi_y \rangle \to \text{constant} > 0 \qquad \text{as } \mathrm{dist}(x, y) \to \infty \tag{2.3}$$

for sufficiently large values of β.

As is (heuristically) discussed in [11], our fractal spin system is also expected to exhibit the long-range order (2.3), for large values of β, if and only if

(i) $Q > 0$ (for spin systems with discrete symmetry, such as Ising [3] and one-component φ^4 models), and

(ii) $d_{\mathrm{SPEC}} > 2$ (for spin systems with continuous symmetry, such as XY, Heisenberg and N-component ($N \geq 2$) φ^4 models), respectively. Here Q is the *connectivity* defined below (see (2.6)) and d_{SPEC} is the *spectral dimension* [2, 6, 7]. It should be noted that both Q and d_{SPEC} are geometric quantities, which are defined independently of the specific spin systems on the lattice.

Moreover it is also conjectured [11] that, in any ferromagnetic spin system,

(iii) $d_{\mathrm{SPEC}} > 4$ implies that the critical phenomena coincide with the mean-field predictions. (More precisely, we expect the critical exponent equalities $\gamma = 1$, $\alpha = 0$, $\Delta_4 = \frac{3}{2}, \ldots$.)

In the ordinary d-dimensional hypercubic lattice, both $Q+1$ and d_{SPEC} coincide with the Euclidean dimension d. Then (i)–(iii) become nothing other than the familiar statements on the *critical dimensionalities.*

Our assertions (i)–(iii) are derived by carefully studying several rigorous and heuristic arguments concerning phase transitions and critical phenomena. Roughly speaking, one may replace the Euclidean dimension d by

(a) the quantity $(Q+1)$ in a statement which relies on the domain wall type argument, and

(b) the spectral dimension d_{SPEC} in a statement which is derived by spin-wave analysis.

2.4. Violation of scaling

Consider a ferromagnetic spin system in the ordinary d-dimensional hypercubic lattice, which undergoes the phase transition. Then it is believed (and partially proven) that, at the *critical point* β_c, the two-point function exhibits the following *power-law decay property*

$$\langle \varphi_x \varphi_y \rangle \sim \mathrm{dist}(x, y)^{-\Delta} \qquad \text{at } \beta = \beta_c.$$

Here the decay exponent Δ is often written as $d-2+\eta$.

In [11], we have shown that, in a fractal spin system, we cannot generally expect such a simple power-law decay. There may be more than one decay mode, characterised by distinct values of the decay exponent. Thus all that we can expect for a fractal spin system at the critical point is

$$\mathrm{dist}(x, y)^{-\Delta'} \leqslant \langle \varphi_x \varphi_y \rangle \leqslant \mathrm{dist}(x, y)^{-\Delta} \qquad \text{at } \beta = \beta_c \tag{2.4}$$

where $\Delta \leqslant \Delta'$. (We have an explicit example [11] where $\Delta \neq \Delta'$.)

Consequently, we suspect that some of the familiar scaling relations, such as Fisher's, are violated in fractal spin systems.

2.5. Rigorous bound for decay exponent

Our main exponent inequality is concerned with the critical decay exponent Δ, which is roughly defined by the right-hand side of (2.4). More precisely, Δ is defined as the supremum value of δ which satisfies the following inequality for all x, y with a finite constant:

$$\langle \varphi_x \varphi_y \rangle \leqslant \text{constant} \times \mathrm{dist}(x, y)^{-\delta} \qquad \text{at } \beta = \beta_c. \tag{2.5}$$

Note that Δ stands for the most slowly decaying mode of the two-point function $\langle \varphi_x \varphi_y \rangle$.

To describe our main result, let us introduce a notion of the *connectivity* [3]. (A rigorous definition appears in § 3.) Consider a region in Λ satisfying $\mathrm{dist}(x_0, x) \leqslant r$ for some fixed origin x_0. Let $N(r)$ be the minimum number of bonds which we must remove from the lattice to isolate the above bounded region from infinity. We assume that $N(r)$ has the following asymptotic behaviour for large values of r:

$$N(r) \sim r^Q \tag{2.6}$$

where Q is a constant which is called the connectivity†.

In the regular lattice, $Q+1$ coincides with the Euclidean dimension. It is quite important to note that this quantity $Q+1$ generally does not coincide with the fractal dimension d_{FRAC}. Finitely ramified fractal lattices (one-dimensional chain, Koch curve, Sierpinski gasket, etc) have various (and arbitrarily large) values of d_{FRAC}, but they all have $Q=0$. For the Sierpinski carpet, we have $d_{\mathrm{FRAC}} = \ln 8/\ln 3 \simeq 1.893$, while $Q = \ln 2/\ln 3 \simeq 0.631$.

Then, from theorem 3 (proved in the next section), we find the following.

† Our definition of Q is slightly different from that in [3]. Our Q does not depend on the embedding of the lattice, while the original Q does. However, two definitions usually coincide if we take a 'natural' embedding.

Corollary 2. In an arbitrary ferromagnetic fractal spin system in the Brydges–Fröhlich–Spencer class, the critical decay exponent Δ and the connectivity Q satisfy the inequality

$$\Delta \leqslant Q. \tag{2.7}$$

The above inequality is proven by showing that the inequality (2.5) with $\delta > Q$ inevitably implies the exponential decay of the correlation function (theorem 3). In the regular lattice, (2.7) reduces to a critical exponent inequality $\eta \leqslant 1$ which is known as the Simon inequality [8].

Our proof, which is a variation of Simon's proof, is unfortunately not its straightforward extension. Simon's original method makes use of the translational invariance of the regular lattice, which is drastically violated in fractals. Instead, we develop a sort of *rigorous renormalisation group* argument which makes use of the fact that a non-trivial fractal has a lot of large holes ('tremas' in the terminology of [1]) which are located in a self-similar manner. Therefore our proof does not apply to regular lattices.

2.6. Discussion

We have presented some rigorous and heuristic results clarifying the important roles of the connectivity Q and the spectral dimension d_{SPEC} in fractal spin systems. It is clear that the behaviour of fractal spin systems is not as simple as those of ordinary spin systems on hypercubic lattices. We have to know *at least* two kinds of dimensions to determine some of the basic features of the phase transitions and critical phenomena.

It should be stressed, however, that our results do not imply that the other notions of dimension [1, 4, 5] are less essential. For example, the fractal (or Hausdorff) dimension d_{FRAC} plays an important role if one considers scale change in the lattice and investigates the scaling limit of a fractal spin system [6]. (Such an investigation is crucial if we consider the field theories on fractal-like spacetime.)

3. Random walks generated by the Simon–Lieb inequality

In this section, which is the technical heart of the present paper, we describe our assumptions and statements explicitly, and prove the inequality (2.7) by employing a novel use of the Simon–Lieb inequality.

First, let us assume that our fractal lattice satisfies the following properties F1–F7 for all $N = 1, 2, \ldots$ with positive constants a, z_1, z_2, $c_1 - c_4$, and $Q(\geqslant 0)$ which are independent of N.

F1. Λ_s is decomposed into a disjoint sum of finite sublattices (not always identical) as $\Lambda_s = \bigcup_i B_i^{(N)}$ where each $B_i^{(N)}$ is called a *block*.

F2. Each block $B_i^{(N)}$ has a linear size of order a^N, i.e.

$$C_1 a^N \leqslant \max_{x,y \in B_i^{(N)}} \mathrm{dist}(x, y) \leqslant C_2 a^N.$$

Definition. The above decomposition of Λ_s determines a set of broken bonds $\tilde{\Lambda}_b^{(N)} = \{(x, y) \in \Lambda_b | x \in B_i^{(N)}, y \in B_j^{(N)}, i \neq j\}$.

F3. $\tilde{\Lambda}_b^{(N)}$ is again decomposed into a disjoint sum as $\tilde{\Lambda}_b^{(N)} = \bigcup_k C_k^{(N)}$ where each $C_k^{(N)}$ is called a *cut.*

F4. The number of bonds in a cut is uniformly bounded by $C_3 a^{NQ}$.

F5. Different cuts in a block $B_i^{(N)}$ are separated at least by a distance of order a^N, i.e.

$$C_4 a^N \leq \min\{\mathrm{dist}(x, y) | x \in (x, x') \in C_k^{(N)}, y \in (y, y') \in C_l^{(N)}, k \neq l\}.$$

Definition. We say that a block $B_i^{(N)}$ and a cut $C_k^{(N)}$ are *neighbouring*, if there is a site in $B_i^{(N)}$ which belongs to a bond in $C_k^{(N)}$.

F6. To each block $B_i^{(N)}$, the number $z_{1,i}^{(N)}$ of neighbouring cuts satisfies $2 \leq z_{1,i}^{(N)} \leq z_1$.

F7. To each cut $C_k^{(N)}$, the number $z_{2,k}^{(N)}$ of neighbouring blocks satisfies $2 \leq z_{2,k}^{(N)} \leq z_2$.

The explicit decompositions into blocks and cuts in simple (but basic) examples can be found in figure 2. In general, the decomposition in the Nth stage is carried out by making 'cuts' through the holes whose linear sizes are of order a^N. (Note that such holes are separated by distances of order a^N.) It is clear that such a decomposition is impossible in the regular lattice.

Note that the property F4 with F6 implies the relation (2.6) where a^N is replaced by r.

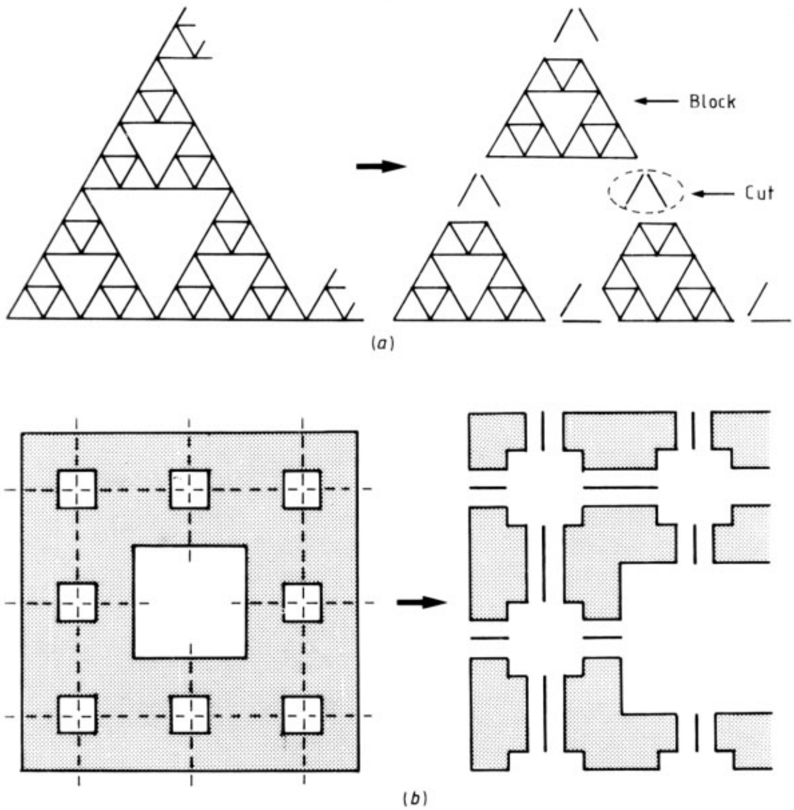


Figure 2. Decomposition of (a) the Sierpinski gasket and (b) the Sierpinski carpet.

Now, to state our main theorem, let us describe in some detail the marginal conditions on the power decay of the two-point functions. Any one of the following three conditions will turn out to imply non-critical exponential decay. (A reader may just take a look at D2 and D3, and then skip to theorem 3.)

For a cut $C_k^{(N)}$ (in F3), we consider a finite sublattice $\Lambda(C_k^{(N)})$ which is obtained by gluing together the blocks (in F1) neighbouring to the cut $C_k^{(N)}$. By $\langle\ldots\rangle_k^{(N)}$, we denote thermal expectation (2.1) defined on this sublattice.

Then the optimal condition for the marginal decay is as follows.

D1. Let $C_l^{(N)}$ and $C_m^{(N)}$ ($l\neq k$, $m\neq k$, $l\neq m$) be the cuts neighbouring the sublattice $\Lambda(C_k^{(N)})$. Then

$$Y^{(N)}=C_3a^{NQ}\max_{k;l,m}\max_{\substack{x\in(x,x')\in C_l^{(N)}\\ y\in(y,y')\in C_m^{(N)}}}\langle\varphi_x\varphi_y\rangle_k^{(N)}$$

converges to zero as $N\to\infty$.

Instead of this rather technical condition, one can use the following two crude (but intuitive) conditions. By applying the assumption F5 and the Griffiths II inequality $\langle\varphi\varphi\rangle^{(N)}\leq\langle\varphi\varphi\rangle$, we see that the following (D2) always implies the above D1.

D2. $\mathrm{dist}(x,y)^Q\langle\varphi_x\varphi_y\rangle\to 0$ holds uniformly in x, y as $\mathrm{dist}(x,y)\to\infty$.

If we restrict ourselves to the finitely ramified fractals ($d_{\mathrm{RAM}}=1$), the following D3 also implies D1.

D3 (only for finitely ramified fractals). $\langle\varphi_{x_0}\varphi_x\rangle\to 0$ holds as $\mathrm{dist}(x_0,x)\to\infty$, where the origin x_0 is the fixed point of the scale transformation [6].

The merit of D3, compared with D2, is that it does not contain any assumptions about the uniformity.

Now we can state our main theorem.

Theorem 3. Consider a ferromagnetic spin system (2.1) in the Brydges-Fröhlich-Spencer class on a fractal lattice Λ satisfying F1-F7. If the two-point function of the system satisfies the condition D1 (or D2, D3), there exist finite constants c, ξ, and

$$0\leq\langle\varphi_x\varphi_y\rangle\leq c\exp(-\mathrm{dist}(x,y)/\xi) \tag{3.1}$$

holds for any x, $y\in\Lambda_s$. Hence the spin system is not at its critical temperature.

Before proving the theorem, we describe how to derive corollary 2 from theorem 3.

Proof of corollary 2. Assume that the system is at its critical point ($\xi=\infty$), and satisfying the bound (2.5) with some $\delta>Q$. Then (2.5) implies the condition D2 and we have a finite ξ, which contradicts the assumption. Thus we obtain the desired inequality (2.7).

If $d_{\mathrm{RAM}}=1$, the existence of *any* decay (condition D3) implies an exponential decay. This reflects the absence of phase transition in the spin systems.

Proof of theorem 3. Recall that, for our spin systems, the following Simon-Lieb inequality is known to hold [8, 9]:

$$\langle\varphi_x\varphi_y\rangle\leq\beta\sum_{\substack{u\in V,v\notin V\\ (u,v)\in\Lambda_b}}\langle\varphi_x\varphi_u\rangle_V\langle\varphi_v\varphi_y\rangle \tag{3.2}$$

where V is a bounded region, $x\in V$, $y\notin V$, and $\langle\ldots\rangle_V$ is the thermal expectation (2.1) defined on V.

Consider the decompositions into blocks and cuts in F1–F7, for a fixed value of N. Let x belong to a cut $C_k^{(N)}$, and a sufficiently separated site y belong to another cut $C_l^{(N)}$. Applying (3.2) to $\langle\varphi_x\varphi_y\rangle$ by setting $V=\Lambda(C_k^{(N)})$, we have

$$\langle\varphi_x\varphi_y\rangle \leq \beta\sum_m \sum_{(u,v)\in C_m^{(N)}} \langle\varphi_x\varphi_u\rangle_k^{(N)}\langle\varphi_v\varphi_y\rangle$$
$$\leq \sum_m \beta Y^{(N)} \max_{(u,v)\in C_m^{(N)}} \langle\varphi_v\varphi_y\rangle \tag{3.3}$$

where the first sum counts those cuts $C_m^{(N)}$ neighbouring $\Lambda(C_k^{(N)})$. We have used F4 and the definition of the weight factor $Y^{(N)}$ follows that of D1.

It is convenient to interpret each term on the right-hand side of (3.3) as describing a 'hop' from a cut $C_k^{(N)}$ to another $C_m^{(N)}$ which passes through a block neighbouring both cuts. Then it is easy to see that successive applications of the Simon–Lieb inequality (3.2) to (3.3) generate 'random walks' which wander from cut to cut around Λ, until they hit the cut $C_l^{(N)}$ containing the site y

$$\langle\varphi_x\varphi_v\rangle \leq \sum_{W:C_k^{(N)}\to C_l^{(N)}} (\beta Y^{(N)})^{n(W)}. \tag{3.4}$$

Here $W=(C_{k_1}^{(N)}, C_{k_2}^{(N)},\ldots, C_{k_{n(W)}}^{(N)})$ where $k_1=k$, $k_{n(W)}=l$, $n(W)$ denotes the number of cuts contained in W, and $C_{k_p}^{(N)}$ and $C_{k_{p+1}}^{(N)}$ are neighbours to a common block $B_{i_p}^{(N)}$ (figure 3). In (3.4), the summation runs over all possible walks satisfying the above conditions.

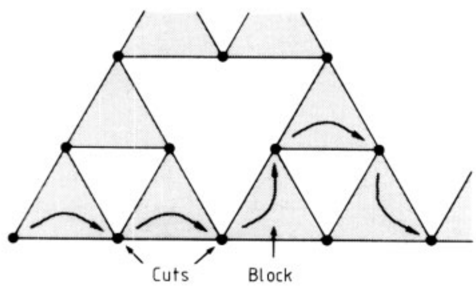


Figure 3. A random walk generated by the Simon–Lieb inequality. A hop from one cut to another is across a block.

Since the number of the possible walks with n cuts is bounded by $(z_1z_2)^n$ from F6 and F7, and since n must be larger than $\mathrm{dist}(x,y)/c_4a^N$ from F5, we obtain the following upper bound:

$$\langle\varphi_x\varphi_y\rangle \leq (1-z_1z_2\beta Y^{(N)})^{-1}\{(z_1z_2\beta Y^{(N)})^{1/C_4a^N}\}^{\mathrm{dist}(x,y)} \tag{3.5}$$

provided that $z_1z_2\beta Y^{(N)}$ is strictly smaller than 1. Since we have (3.4) and D1, the condition can be satisfied by taking sufficiently large N. Then (3.5) implies the desired exponentially decaying upper bound (3.1).

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